Physics 3
Problems II.
Solid State Physics.

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The problems presented here together with their solutions help the deeper understanding of the lectures. Similar problems may be given in the tests. We tried to give a very detailed solution for every one of them, however in the tests you need not give so much details. This material will be refreshed from time to time, so please check the date on the title page!
Chapter 2

Solid State Physics

2.1 Formulas used in the solutions

There exists an algebraic formula between the $b_1$, $b_2$, $b_3$ primitive reciprocal lattice vectors and the three primitive direct lattice vectors $a_1$, $a_2$, $a_3$

\[
\begin{align*}
    b_1 &= 2\pi \frac{a_2 \times a_3}{a_1 \cdot (a_2 \times a_3)} \\
    b_2 &= 2\pi \frac{a_3 \times a_1}{a_1 \cdot (a_2 \times a_3)} \\
    b_3 &= 2\pi \frac{a_1 \times a_2}{a_1 \cdot (a_2 \times a_3)}
\end{align*}
\]  
(2.1.1)

The effective masses of electrons and holes in Si and Ge are in Table 2.1. The effective mass for conductivity of electrons $m_{cc}$ is what we must use for mobility or diffusion constant calculations. For cubic isotropic semiconductors with anisotropic dispersion relations minima (again e.g. Si and Ge), one has to sum over the effective masses in the different minima along the equivalent directions. The resulting effective mass for bands which have ellipsoidal constant energy surfaces is given by:

\[
m_{cc} = \frac{3}{\frac{1}{m_l} + \frac{1}{m_t} + \frac{1}{m_t}}
\]

The general formula from the calculations above can be expressed in the simple form

\[
\begin{align*}
    N_e(T) &= 2.5 \times 10^{19} \left( \frac{m_e}{m_e} \right)^{\frac{3}{2}} \left( \frac{T}{300K} \right)^{\frac{3}{2}} \\
    P_v(T) &= 2.5 \times 10^{19} \left( \frac{m_h}{m_e} \right)^{\frac{3}{2}} \left( \frac{T}{300K} \right)^{\frac{3}{2}}
\end{align*}
\]  
(2.1.2, 2.1.3)
Table 2.1: Effective masses in Si and Ge

<table>
<thead>
<tr>
<th></th>
<th>rel. eff.mass</th>
<th>Si</th>
<th>Ge</th>
</tr>
</thead>
<tbody>
<tr>
<td>longitudinal</td>
<td>( m_l/m_e )</td>
<td>0.98</td>
<td>1.59</td>
</tr>
<tr>
<td>transversal</td>
<td>( m_t/m_e )</td>
<td>0.19</td>
<td>0.0815</td>
</tr>
<tr>
<td>dens. of states</td>
<td>( m_e/m_e )</td>
<td>0.36</td>
<td>0.22</td>
</tr>
<tr>
<td>conduct.</td>
<td>( m_{ce}/m_e )</td>
<td>0.26</td>
<td>0.12</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>rel. eff.mass</th>
<th>Si</th>
<th>Ge</th>
</tr>
</thead>
<tbody>
<tr>
<td>heavy</td>
<td>( m_{h}/m_e )</td>
<td>0.49</td>
<td>0.33</td>
</tr>
<tr>
<td>light</td>
<td>( m_{lp}/m_e )</td>
<td>0.16</td>
<td>0.043</td>
</tr>
<tr>
<td>split-off band</td>
<td>( m_{so}/m_e )</td>
<td>0.24</td>
<td>0.084</td>
</tr>
<tr>
<td>dens. of states</td>
<td>( m_{v}/m_e )</td>
<td>0.81</td>
<td>0.34</td>
</tr>
</tbody>
</table>

As a result of collisions the ordered part of the electron velocities, called the drift velocity will be constant and proportional with the field strength:

\[
\mathbf{v}_{drift} = \mu \mathbf{E}
\]  

where the constant of proportionality is the mobility \( \mu \) of the electrons

\[
\mu = \left( \frac{e \tau}{m_e} \right)
\]

\[
\mathbf{j}(=\sigma \mathbf{E}) = \left( \frac{ne^2 \tau}{m_e} \right) \mathbf{E}
\]
and

\[ \sigma = \frac{ne^2\tau}{m_e} \]  \hspace{1cm} (2.1.6)

One way to determine the value of \( \tau \) is from (2.1.6), where the density \( n \) of conduction electrons is

\[ n = L_A \frac{Z \rho_m}{A} \]  \hspace{1cm} (2.1.7)

\[ j = e(n_e \mu_e + p_e \mu_h)E \]  \hspace{1cm} (2.1.8)

\[ j = n_i e \langle v \rangle = n_i e(\mu_e + \mu_h)E \quad \text{and} \quad j = \sigma E \]

\[ \sigma = n_i e(\mu_e + \mu_h) \]  \hspace{1cm} (2.1.9)

<table>
<thead>
<tr>
<th>Metal</th>
<th>Li</th>
<th>Na</th>
<th>K</th>
<th>Rb</th>
<th>Cs</th>
<th>Cu</th>
<th>Ag</th>
<th>Au</th>
<th>Mg</th>
<th>Al</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mathcal{E}_F(\text{eV}) )</td>
<td>4.7</td>
<td>3.1</td>
<td>2.1</td>
<td>1.8</td>
<td>1.5</td>
<td>4.1</td>
<td>5.5</td>
<td>5.5</td>
<td>7.3</td>
<td>11.9</td>
</tr>
</tbody>
</table>

**Table 2.2:** Fermi energies of some metals

Any of these energy states can be occupied by 2 electrons with opposite spins. If each atom contribute \( s \) conduction electrons to the crystal then at \( T = 0 \text{K} \) all possible energy states below

\[ \mathcal{E}_F (= \mathcal{E}_{\text{max,occupied}}) = \frac{\hbar^2 \pi^2 s}{2m_e a^2} \]  \hspace{1cm} (2.1.10)

will be occupied and all levels above it will be empty. \( \mathcal{E}_F \) is called the Fermi energy of the system. We denote the corresponding \( k \) value with \( k_F \) and call it the Fermi wave vector.

The Fermi-Dirac distribution function:

\[ f_{FD}(\mathcal{E}) = \frac{1}{e^{(\mathcal{E}_F-\mathcal{E})/k_B T} + 1} \]  \hspace{1cm} (2.1.11)

The number of electrons \( (N_e = N \cdot s) \):

\[ N_e = 2 \frac{k_F^3}{6\pi^2} V \quad \Rightarrow \quad n(\equiv \frac{N_e}{V}) = \frac{k_F^3}{3\pi^2} \]  \hspace{1cm} (2.1.12)
When \( s = 2 \) the length of the Fermi wave vector will be \( 2\pi/a \) and the allowed \( k \) values will satisfy

\[ k \in \left[ -\frac{\pi}{a}, \frac{\pi}{a} \right] \]

For n-type semiconductors \( (N_d^+ > N_a^- \text{ where } N_d^+ \approx N_d \text{ and } N_a^+ \approx N_a) \):

\[ n_c \approx (N_d - N_a) \quad p_v \approx \frac{n_i^2}{N_d - N_a} \quad (2.1.13) \]

while for p-type semiconductors \( (N_a > N_d) \):

\[ p_v \approx (N_a - N_d) \quad n_c \approx \frac{n_i^2}{N_a - N_d} \quad (2.1.14) \]

\[ n_c(T) = N_c e^{-\frac{\epsilon_F - \epsilon_p}{k_B T}} \quad (2.1.15a) \]

\[ p_v(T) = P_v e^{-\frac{\epsilon_F - \epsilon_v}{k_B T}} \quad (2.1.15b) \]

where \( N_c \) and \( P_v \) are the effective density of states in the conduction and valence bands respectively:

\[ N_c(T) = 2 \cdot \left( \frac{2\pi m_e k_B T}{h^2} \right)^{\frac{3}{2}} \quad (2.1.16) \]

\[ P_v(T) = 2 \cdot \left( \frac{2\pi m_h k_B T}{h^2} \right)^{\frac{3}{2}} \quad (2.1.17) \]

\[ d_n = \sqrt{\frac{N_a}{N_d(N_d + N_a)} \frac{\epsilon \Delta \phi}{2e}} \quad (2.1.18) \]

\[ d_p = \sqrt{\frac{N_d}{N_a(N_d + N_a)} \frac{\epsilon \Delta \phi}{2e}} \]

In doped semiconductors:

\[ n_c^{(d)}(T) = n_i e^{-\frac{\epsilon_F^{(d)} - \epsilon_p}{k_B T}} \quad \text{for an n-type semiconductor} \quad (2.1.19a) \]

\[ p_v^{(a)}(T) = n_i e^{-\frac{\epsilon_F^{(a)} - \epsilon_v}{k_B T}} \quad \text{for a p-type semiconductor} \quad (2.1.19b) \]
\[ \mathcal{E}_F^{(d)} = \frac{1}{2} (\mathcal{E}_c + \mathcal{E}_{\text{donor}}) + \frac{3}{4} k_B T \ln \left( \frac{m_h}{m_e} \right) \]  
\[ \mathcal{E}_F^{(a)} = \frac{1}{2} (\mathcal{E}_v + \mathcal{E}_{\text{acceptor}}) + \frac{3}{4} k_B T \ln \left( \frac{m_h}{m_e} \right) \]  

\[
E \Delta \varphi = k_B T \ln \left( \frac{N_d N_a}{n_i^2(T)} \right) \quad \text{ (2.1.21)}
\]

For intrinsic semiconductors

\[ n_e(T) = p_v(T) \quad \text{ (2.1.22)} \]

\[ n_i \equiv n_e(T) = p_v(T) = \sqrt{N_c(T)P_v(T)} e^{-\frac{E_g}{k_B T}} \quad \text{ (2.1.23)} \]

---

Table 2.3: Shallow donor and acceptor levels in Si and Ge in eV.

<table>
<thead>
<tr>
<th>acceptor in</th>
<th>B</th>
<th>Al</th>
<th>Ga</th>
<th>In</th>
<th>Te</th>
</tr>
</thead>
<tbody>
<tr>
<td>Si</td>
<td>0.046</td>
<td>0.057</td>
<td>0.065</td>
<td>0.16</td>
<td>0.26</td>
</tr>
<tr>
<td>Ge</td>
<td>0.01</td>
<td>0.01</td>
<td>0.061</td>
<td>0.011</td>
<td>0.01</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>donor in</th>
<th>P</th>
<th>As</th>
<th>Sb</th>
<th>Bi</th>
</tr>
</thead>
<tbody>
<tr>
<td>Si</td>
<td>0.046</td>
<td>0.057</td>
<td>0.065</td>
<td>0.16</td>
</tr>
<tr>
<td>Ge</td>
<td>0.01</td>
<td>0.01</td>
<td>0.061</td>
<td>0.011</td>
</tr>
</tbody>
</table>

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2.2 Problems

Problem 1.
Enumerate the symmetries the following “crystal” has.

Solution:
This “crystal” has the following symmetry elements:

- 1 - 4-fold rotation axis ($A_4$)
- 4 - 2-fold rotation axes ($A_2$), 2 cutting the faces and 2 cutting the edges
- 5 mirror planes (m), 2 cutting across the faces, 2 cutting through the edges, and one cutting horizontally through the center.
- There is a center of symmetry (i).

Problem 2.
Calculate the surface density of atoms in a bcc crystal if the lattice constant is $a = 0.5 \, nm \, (= \, 5) \AA$ and the surface plane cuts the cells diagonally and it is perpendicular to the plane of the $a_1$ and $a_2$ vectors.

Solution:
The plane in question goes through 4 corner atoms and the middle atom of the cell. Only 1/4th of each of the cross sections of the corner atoms belong to our cell, while the cross section of the middle atom is completely inside it. Therefore the number of atoms on this plane is 2. The area of the plane is \( a \cdot a\sqrt{2} \), so the density of atoms is

\[
\frac{2}{a^2\sqrt{2}} = \frac{\sqrt{2}}{(0.5\times10-9)^2} = 5.66 \times 10^{18} \text{atoms m}^{-2}
\]

\( ^a \)i.e. it is a (110) plane - see Miller indices below.

**Problem 3.**

With (2.1.1) we can easily prove that the reciprocal lattice of an fcc lattice is a bcc lattice:

Start with the following selection of primitive fcc lattice vectors:
Then the 3 primitive vectors are

\[
\begin{align*}
\mathbf{a}_1 &= \frac{a}{2}(\mathbf{i} + \mathbf{k}) \\
\mathbf{a}_2 &= \frac{a}{2}(\mathbf{i} + \mathbf{j}) \\
\mathbf{a}_3 &= \frac{a}{2}(\mathbf{j} + \mathbf{k})
\end{align*}
\]

(2.2.1)

Determine first the denominator in (2.1.1), which is the volume of the primitive cell:

\[
\mathbf{a}_1 \cdot (\mathbf{a}_2 \times \mathbf{a}_3) = \frac{a^3}{8} (\mathbf{i} + \mathbf{k}) \cdot ((\mathbf{i} + \mathbf{j}) \times (\mathbf{j} + \mathbf{k}))
\]

\[
= \frac{a^3}{8} [\mathbf{i} \cdot (\mathbf{i} \times \mathbf{j}) + \mathbf{i} \cdot (\mathbf{i} \times \mathbf{k}) + \mathbf{i} \cdot (\mathbf{j} \times \mathbf{k}) + \mathbf{k} \cdot (\mathbf{i} \times \mathbf{j}) + \mathbf{k} \cdot (\mathbf{i} \times \mathbf{k}) + \mathbf{k} \cdot (\mathbf{j} \times \mathbf{k})]
\]

\[
= \frac{a^3}{8} (\mathbf{i} \cdot (\mathbf{j} \times \mathbf{k}) + \mathbf{k} \cdot (\mathbf{i} \times \mathbf{j}))
\]

here we used that \(\mathbf{i}, \mathbf{j}\) and \(\mathbf{k}\) are perpendicular to each other. Furthermore

\[
\mathbf{i} \times \mathbf{j} = \mathbf{k}, \quad \mathbf{j} \times \mathbf{k} = \mathbf{i} \quad \text{and} \quad \mathbf{k} \times \mathbf{i} = \mathbf{j}
\]

Therefore

\[
\mathbf{a}_1 \cdot (\mathbf{a}_2 \times \mathbf{a}_3) = \frac{a^3}{4}
\]

Now work with the numerators using the same formulas for \(\mathbf{i}, \mathbf{j}\) and \(\mathbf{k}\):

\[
\mathbf{a}_2 \times \mathbf{a}_3 = \frac{a^2}{4}(\mathbf{k} - \mathbf{j} + \mathbf{i})\mathbf{a}_3 \times \mathbf{a}_1 = \frac{a^2}{4}(\mathbf{i} - \mathbf{k} + \mathbf{j})\mathbf{a}_1 \times \mathbf{a}_2 = \frac{a^2}{4}(\mathbf{j} - \mathbf{i} + \mathbf{k})
\]

Which gives us the reciprocal base vectors:

\[
\begin{align*}
\mathbf{b}_1 &= \frac{2\pi}{a} \frac{a^2}{4}(\mathbf{k} - \mathbf{j} + \mathbf{i}) \\
\mathbf{b}_2 &= \frac{2\pi}{a} (\mathbf{i} - \mathbf{k} + \mathbf{j}) \\
\mathbf{b}_3 &= \frac{2\pi}{a} (\mathbf{j} - \mathbf{i} + \mathbf{k})
\end{align*}
\]

(2.2.2)

Compare (2.2.2) with vectors in Fig. 2.1 to see that we, in fact got the primitive vectors of a bcc lattice. The only difference is that the length is now \(\frac{2\pi}{a}\). This means that the volume of the primitive cell in a reciprocal lattice is \(\frac{(2\pi)^3}{V}\) where \(V\) is the volume of the primitive cell of the original lattice.
**Problem 4.**
Determine the Miller indices for the plane on the figure!

![Image of a crystal lattice with Miller indices](image)

**Solution:**
The intersections with the three axes are at $4a_1$, $3a_2$ and $2a_3$. Then the inverse intercepts in lattice vector units are:

\[
\frac{1}{4}, \frac{1}{3}, \frac{1}{2}
\]

To get integer numbers we have to calculate the lowest common denominator of this fraction, which is 12. Multiplying each fraction with 12 gives the three Miller indices: \((346)\)

---

**Problem 5.**
In the previous example let the length of all the three base vectors of the direct lattice $a = 5\text{nm}$. Determine the distance of the lattice planes \((346)\).

**Solution:**
The three basis vectors are of the same length and they are perpendicular to each other. Therefore the three reciprocal base vectors will also be of the same length and perpendicular to each other. By substitution into (2.1.1) this length
is \( b = \frac{2 \pi}{a} = 1.26 \text{ nm}^{-1} \). The length of the reciprocal vector perpendicular to the \((346)\) planes is

\[
|b_{346}| = \sqrt{3^2 + 4^2 + 6^2} \cdot \frac{2 \pi}{a} \left( = \sqrt{61} \cdot 1.26 = 9.81 \text{ nm}^{-1} \right)
\]

The distance of the planes \((346)\) therefore is

\[
d_{346} = \frac{a}{\sqrt{61}} = 0.64 \text{ nm}
\]

Problem 6.

Draw all 9 lattice planes and determine the Miller indices in a simple cubic Bravais lattice.

Solution:

(Where is the origin of the 3 lattice vectors in the cubes?)

Problem 7.

Determine the possible diffraction angles for an X-ray of 10 keV from the \((111)\) planes of a simple cubic lattice, if the lattice constant is \( a = 5.3 \text{ Å} \).

Solution:
The wavelength of the X-ray:

\[ E = h \nu = \frac{hc}{\lambda} \Rightarrow \lambda = \frac{hc}{E} = 1.24 \times 10^{-10} \text{m} \]

To apply Bragg’s law

\[ 2d \sin \theta = n \lambda \]

we need to calculate the distance of the lattice planes of Miller indices (111), which are planes going through 3 non adjacent corner of the cube. The distance of two such planes that intersects a primitive cell (see the figure in the previous example) is 3rd of the body diagonal:

\[ d = \frac{1}{3} \sqrt{3} a = 0.305 \text{nm} \]

The possible diffraction angles are determined by:

\[ \sin \theta = n \frac{\lambda}{2d} = 0.20325 n \]

Here \( n = 1, 2, 3, 4 \), i.e. the angles are

11.72°, 23.99°, 3.57°, and 54.39°
Problem 8.

Aluminum has three valence electrons per atom, an atomic weight of 0.02698 kg/mol, a density of 2700 kg/m$^3$, and a conductivity of $3.54 \times 10^7$ S/m. Calculate the electron mobility in aluminum. Assume that all three valence electrons of each atoms are free.

Solution:

The number of aluminum atoms per m$^3$ is

$$n_a = \frac{6.0210^{23} \text{ atoms/mol}}{1/0.02698 \text{ mol/kg}} \cdot \frac{1}{2700 \text{ kg/m}^3} = 6.024 \times 10^{28} \text{ atoms/m}^3$$

Thus the electron density in aluminum is

$$n = 3 \cdot 6.024 \times 10^{28} \text{ atoms/m}^3 = 1.807 \times 10^{29} \text{ electron/m}^3$$

From (2.1.5)

$$\mu = \frac{\sigma}{ne} = \frac{3.54 \times 10^7}{1.807 \times 10^{29} \cdot 1.6022 \times 10^{-19}} = 1.22 \times 10^{-3} \text{ m/s}$$
Problem 9.
What is the value of $\tau$ in silver at $t = 0^\circ C$ if the measured resistivity is $1.51 \cdot 10^{-8} \Omega m$?

Solution:
From (2.1.6), (2.1.7) using the definition of $\rho$

\[ \tau = \frac{m_e}{\rho(T) n e^2} = \frac{m_e A}{L_A Z \rho_m e^2 \rho} \]

Silver has a single $5s^1$ electron so $Z = 1$ and the mass density is $\rho_m = 10.49 g/cm^3 = 1.049 \cdot 10^4 kg/m^3$, $A = 107.8682 g/mol = 0.1078682 kg/mol$ and $\rho(273 K) = 1.51 \cdot 10^{-8} \Omega m$. After substitution

\[ \tau = 4.013 \cdot 10^{-14} s \]

Problem 10.
What is the quasi-free electron density in copper? Calculate the Fermi velocity and momentum too

Solution:
From Table 2.2 and formulas (2.1.12) and (2.1.10)

\[ n = \frac{(2m_e E_F)^{3/2}}{2 \pi^2 \hbar^3} = 5.655 \cdot 10^{28} \text{electron/m}^3 \]

\[ v_F = 1.2 \cdot 10^6 m/s = 0.004 c \quad k_F = 1.0 \cdot 10^{11} 1/m \]

Problem 11.
The dispersion relation of electrons in the valence and conduction bands near the band edges are approximated by the following functions:

\[ E_v(k) = -3.024 \cdot 10^{-20}(k - 2.4510^8)^2 + 13 \quad [eV] \]

\[ E_c(k) = 4.65 \cdot 10^{-20} k^2 + 11.9 \quad [eV] \]

Express the effective masses of electrons in units of the free electron mass $m_e = 9.1 \cdot 10^{-31} kg$.

Solution:
In 1D
\[ \frac{1}{m_{\text{eff}}} = \frac{1}{\hbar^2} \frac{d^2 \mathcal{E}(k)}{dk^2} \]

The energies in eV converted to Joule are:
\[ \mathcal{E}_v(k) = -4.845 \times 10^{-39}(k - 2.4510^8)^2 + 2.08 \cdot 10^{-18} \quad [\text{J}] \]
\[ \mathcal{E}_c(k) = +7.450 \times 10^{-39}k^2 + 1.91 \cdot 10^{-18} \quad [\text{J}] \]

The second derivative of both function gives twice the coefficient of the 2nd order term, and so the electron effective masses in the conduction and valence bands are:
\[ m_{\text{eff}}^{(v)} = -\frac{\hbar^2}{1.490 \times 10^{-38}} = -7.46 \cdot 10^{-31}[\text{kg}] = -0.819 \, m_e \]
\[ m_{\text{eff}}^{(c)} = \frac{\hbar^2}{9.690 \times 10^{-39}} = 1.14 \cdot 10^{-30}[\text{kg}] = 1.260 \, m_e \]

**Problem 12.**

Comparing the electron excitation probabilities in silicon at room temperature and at \( T = 450 \, K \).

**Solution:**

Substituting into the probability formula: at room temperature
\[ \mathcal{P}(300K) = 3.32 \times 10^{-19} \]

while at \( T = 450K \):
\[ \mathcal{P}(450K) = 4.79 \times 10^{-13} \]

So the ratio of these probabilities is
\[ \frac{\mathcal{P}(450K)}{\mathcal{P}(300)} = 1.44 \times 10^6 \]

i.e. the number of electrons available for conduction is about \( 10^6 \) larger at 450K than at 300 K. In the same range the increase in the resistivity because of lattice vibrations is about linear. As a consequence resistivity of Si decreases with increasing temperature, i.e. has a negative thermal coefficient contrary to metals. The same is true for all semiconductors.
Problem 13.
A rod of intrinsic Si is 1 cm long and has a diameter of 1mm. At room temperature, the intrinsic concentration in the silicon is \( n_i = 1.5 \times 10^{16} \text{m}^{-3} \). The electron and hole mobilities are \( \mu_e = 0.13 \text{m}^2\text{V}^{-1}\text{s}^{-1} \) and \( \mu_h = 0.05 \text{m}^2\text{V}^{-1}\text{s}^{-1} \). Calculate the conductivity \( \sigma \) of the silicon and the resistance \( R \) of the rod.

Solution:
\[
\sigma = n_i e (\mu_e + \mu_h) = 4.33 \times 10^{-4} \Omega \text{m}
\]
\[
R = \frac{l}{\sigma d^2 \pi / 4} = 29.4 \text{M} \Omega
\]

Problem 14.
Calculate the effective carrier densities of states in the conduction and valence bands of germanium and silicon at 300 K.

Solution:
Substitute the effective masses for the density of states from Table 2.1 into (2.1.16) and (2.1.17).

<table>
<thead>
<tr>
<th>( cm^{-3} )</th>
<th>Ge</th>
<th>Si</th>
</tr>
</thead>
<tbody>
<tr>
<td>( N_c(300K) )</td>
<td>1.02 ( 10^{19} )</td>
<td>2.81 ( 10^{19} )</td>
</tr>
<tr>
<td>( P_v(300K) )</td>
<td>5.64 ( 10^{18} )</td>
<td>1.83 ( 10^{19} )</td>
</tr>
</tbody>
</table>

Problem 15.
A rod of n-type extrinsic Si is 1 cm long and has a diameter of 1mm. At room temperature, the donor concentration is \( 5 \times 10^{14} \text{atom/cm}^3 \) and this corresponds to 1 impurity for \( 10^8 \) Si atoms. A steady 2\( \mu \text{A} \) current is flowing through the bar. Determine the electron and hole concentration, the conductivity and the voltage across the rod. The intrinsic electron concentration in silicon is \( n_i = 1.01 \times 10^{10} \text{m}^{-3} \). The electron mobility is \( \mu_e = 0.13 \text{m}^2\text{V}^{-1}\text{s}^{-1} \).

Solution:
For n-type: \( N_a \approx 0 \). From (2.1.13) \( n = N_d = 5 \times 10^{20} / \text{m}^3 \),

\[
p_v = \frac{n_i^2}{N_d} = 4.5 \times 10^{13} / \text{m}^3
\]

From (2.1.8) and using that \( p_v \ll n_c \)

\[
\sigma = e n_c \mu_e = 0.104 \text{S/m}
\]
The voltage across the rod:

\[ U = \frac{I L}{\sigma A} = 0.12 \text{ V} \]

\[ ^{a}\text{The previously accepted value before 1991 was } 1.45 \cdot 10^{10} \]
Problem 16.

Determine the ratio of conduction electrons from P dopants in Si to the intrinsic electron concentration at the following temperatures: room temperature, 100°C and 500°C! Is it possible for the intrinsic electron concentration to become larger than the one due to the dopants?

Solution:

a) P is a donor atom, therefore the ratio of the conduction electrons from P and from the valence band can be calculated according to (2.1.19), using (2.1.20a) and neglecting the factor $\ln \left( \frac{m_c}{m_h} \right)$:

$$\frac{n_c^{(d)}(T)}{n_i(T)} = e^{-\frac{\mathcal{E}_d - \mathcal{E}_c}{k_B T}}$$

$$= e^{-\left( \frac{\mathcal{E}_d - \mathcal{E}_c}{k_B T} \right)}$$

$$= e^{-\left( \frac{\mathcal{E}_d - \mathcal{E}_c + \mathcal{E}_g}{k_B T} \right)}$$

The value of $\mathcal{E}_d - \mathcal{E}_c$ from Table 2.3 is $-0.046$ eV, $\mathcal{E}_g = 1.12$ eV, and $k_B T$ at room temperature (300 K) equals to 0.0258 eV so

$$\frac{n_c^{(d)}(T)}{n_i(T)} = e^{-\left( \frac{1.12 - 0.046}{0.0258} \right)} = 1.09 \cdot 10^9$$

Similarly at 100°C (373K) and 500°C (773K)

$$\frac{n_c^{(d)}(373K)}{n_i(373K)} = 1.08 \cdot 10^7$$

$$\frac{n_c^{(d)}(773K)}{n_i(773K)} = 3.17 \cdot 10^3$$

The conduction electron concentration in P doped Si at room temperature is $\approx 10^9$ times larger than the electron concentration in intrinsic Si, and about a thousand times as large even at the very high temperature of 737 K!
From this formulas it seems that the intrinsic electron concentration may never reach \( n_i^{(d)} \) and the two concentrations may never even be equal. But this is not so. We made some assumptions, which become invalid at higher temperatures. First we neglected the logarithmic terms when we calculated the Fermi energies and second we neglected the fact that the donor concentration is very small and the number of conduction electrons from donors has an upper limit, i.e. the electron density due to dopants has *saturation*. Therefore it is possible for the intrinsic electron concentration to exceed the electron concentration from dopants.

**Problem 17.**

Calculate the total voltage difference (the *built in potential*) between the n-type and p-type part for a uniformly doped Silicon p-n junction with \( N_d = N_a = 10^{17} \text{cm}^{-3} \) at room temperature. The intrinsic carrier density is \( 1.45 \times 10^{14} \text{m}^{-3} \). Will the built-in voltage increase or decrease with an increase in temperature?

**Solution:**

From formula (2.1.21)

\[
V_{p-n} \equiv \Delta \varphi = \frac{1}{e} k_B T \ln \left( \frac{N_d N_a}{n_i^2} \right) \quad (2.2.3)
\]

\[
V_{p-n} = 1.38 \times 10^{-23} \left[ \frac{J}{K} \right] 300[K] \frac{1}{1.6022 \times 10^{-19}[\text{As}]} \ln \left( \frac{10^{21} \times 10^{21}}{(1.45 \times 10^{14})^2} \right)
\]

\[
= 0.82 \text{V}
\]

Substituting back the expression (2.1.23) of \( n_i^2(T) \) and (2.1.3) we find that \( V_{p-n} \) is of the form:

\[
e V_{p-n} = const_1 k_B T - const_2 k_B T \ln T + \mathcal{E}_g
\]

When \( T \) increases the change in the term containing \(-k_B T \ln T\) is larger than the change in the term containing \( k_B T \to \) the voltage decreases.

**Problem 18.**

Determine the widths \( d_n \) and \( d_p \) and the electrical field strength for the Si p-n junction of the previous example. The relative permittivity of Si is \( \epsilon_r = 16.0 \).

**Solution:**
We may write (2.1.18) in a more convenient form:

\[
d_n = 5257 \sqrt{\frac{N_a}{N_d (N_d + N_a)}} \frac{\epsilon_r \Delta \varphi}{2} [\text{nm}] \\
d_p = 5257 \sqrt{\frac{N_d}{N_a (N_d + N_a)}} \frac{\epsilon_r \Delta \varphi}{2} [\text{nm}]
\]

Substituting the data from the previous example gives \(d_n = d_p = 425.8 \text{ nm}\).

The magnitude of \(E\) is \(\Delta \varphi/(d_n + d_p) = 0.82V/4.258 \times 10^{-7} \text{ m} = 1.93 \times 10^7 \text{ V/m}\). 

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